How many symbols for k-Thue sequences?

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A sequence is called *nonrepetitive* or *Thue* if it does not contain a repetition of any length. Currie and Simpson introduced a generalization of this notion. A sequence S is k-*Thue* if every j-subsequence of S is Thue, for $1 \leq j \leq k$. Here, a j-*subsequence* of S is a subsequence $\xi_i \xi_{i+j} \xi_{i+2j} \ldots$, for any i.

In 2002 Grytczuk conjectured that $k + 2$ symbols are enough to construct a k-Thue sequence of arbitrary length and it was shown that the conjecture is true for $k = 2, 3$ and 5.

We present a construction of arbitrarily long k-Thue sequences on $2k$ symbols, which improves the previous bound of $2k + 10\sqrt{k}$. Moreover, we confirm the conjecture for the cases $k = 4$ and 6 in two ways and present constructions of both cases using 6 and 8 symbols, respectively.